A social network analysis trust-consensus based approach to group decision-making problems with interval-valued fuzzy reciprocal preference relations☆

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Abstract

A social network analysis (SNA) trust-consensus based group decision making model with interval-valued fuzzy reciprocal preference relation (IFRPR) is investigated. The main novelty of this model is that it determines the importance degree of experts by combining two reliable resources: trust degree (TD) and consensus level (CL). To do that, an interval-valued fuzzy SNA methodology to represent and model trust relationship between experts and to compute the trust degree of each expert is developed. The multiplicative consistency property of IFRPR is also investigated, and the consistency indexes for the three different levels of an IFRPR are defined. Additionally, similarity indexes of IFRPR are defined to measure the level of agreement among the group of experts. The consensus level is derived by combining both the consistency index and similarity index, and it is used to guide a feedback mechanism to support experts in changing their opinions to achieve a consensus solution with a high degree of consistency. Finally, a quantifier guided non-dominance possibility degree (QGNDPD) based prioritisation method to derive the final consensus-trust based solution is proposed.

Keywords: Decision Making, Interval-valued Fuzzy Reciprocal Preference Relations, Social Network Analysis, Trust Degree, Consensus.

1. Introduction

In the procedure of group decision-making (GDM), experts usually need to compare a finite set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ with respect to a single criterion, and construct preference relations. In general, there are two basic preference relations: multiplicative preference relation [3, 35, 39] and fuzzy preference relation [4, 33]. In both cases, the preference relation elements represent the dominance of one alternative over another and take the form of exact numerical values. However, many decision making processes take place in an environment in which the information is not precisely

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known [1, 12, 12, 20, 34, 36, 46, 50, 53]. As a consequence, experts may feel more comfortable using an interval number rather than an exact crisp numerical value to represent their preference. Therefore, interval-valued fuzzy reciprocal preference relations (IFRPRs) [22, 49] can be considered an appropriate representation format to capture experts' uncertain preference information. Indeed, the use of IFRPRs in GDM problems under uncertain environments has recently attracted the attention of many researchers [13, 30, 41, 51].

In GDM problems, the individual preferences are aggregated to a collective one for deriving a solution. This is achieved by determining aggregation weights for each expert to compute the collective preference of the group from the individual preferences. As a consequence, one key issue that needs to be addressed in this type of decision making environment is how "weights of experts" should be derived. In most GDM models, the weights of expert are usually considered to be known beforehand or provided by a reliable source being therefore no part of the decision model design. However, in some cases, these assumptions may be unrealistic or improbable. Thus, it could be interesting to provide alternative ways to obtain such information.

Trust can reflect the actual reputation between experts [2] because it uses the history of an expert's actions or behaviour. Therefore, it should be taken into account as a reliable source to be used in deriving aggregation weights for individual experts. Social Network Analysis (SNA) methodology studies the relationships between social entities like members of a group, corporations or nations and it is a useful methodology to examine structural and locational properties such as: centrality, prestige and structural balance [18, 37, 40]. In this article, we focus on one type of social networks in which the users explicitly express their opinion on other users as trust statements. Furthermore, to represent the uncertainty or fuzziness of trust relationship between group experts, this article develops an interval-valued fuzzy SNA to define and measure the trust degree (TD) of individual experts.

Additionally to TD, consensus level (CL) has been previously considered another reliable source to derive the weights for individual experts in consensus models [5, 7, 26, 42–45, 47, 49]. However, these consensus models are static in nature because they do not produce any type of rules to increase consensus when it is unacceptably low. Obviously, it is preferable that the group of experts achieve a high consensus level before aggregating individual preferences into a collective one. Recently, Chiclana et al. [8] and Herrera-Viedma et al. [25] investigated methodologies to develop feedback mechanisms to produce recommendations on how to increase consensus level. Inspired by these approaches, new consensus level (CL) and feedback mechanism for GDMs with IFRPRs are proposed.

Combining the two reliable sources representing the importance degree of experts, the trust degree (TD) and the consensus level (CL), a trust-consensus based approach to determine the weights of experts to use in aggregating individual IFRPRs into the collective one is proposed. Then, by applying the possibility degree (PD) of interval-valued fuzzy numbers (IFNs), a quantifier guided non-dominance possibility degree (QGNDPD) method is developed to derive the priority vector of the collective

IFRPR.

The rest of paper is set out as follows: Section 2 introduces the multiplicative transitivity property and the corresponding definition of consistency for IFRPRs. In Section 3, the trust degree (TD) of experts is computed using SNA. A consensus model for GDM with IFRPRs is presented in Section 4, with special attention paid to the design of the consistency-consensus based feedback mechanism. Section 5 develops a process for deriving the collective IFRPR via the aggregation of the individual IFRPRs that is driven by a trust-consensus based methodology to determine the weights of experts. A quantifier guided non-dominance possibility degree (QGNDPD) method to exploit the collective IFRPR is also presented in this section. An analysis of the trust-consensus based model with respect to other GDM models is proposed in Section 6. Finally, conclusions are drawn in Section 7.

2. Consistency of Interval-Valued Fuzzy Reciprocal Preference Relations

Let X be a universe of discourse. A fuzzy set A on X is characterised by a membership function $\mu_A : X \to [0,1]$, and it is expressed as follows [53]:

$$A = \{(x, \mu_A(x));\ \mu_A(x) \in [0,1]\ \forall x \in X\}. \tag{1}$$

Note that the membership grades of A are crisp numbers.

Given three alternatives x_i, x_j, x_k such that x_i is preferred to x_j and x_j to x_k, the question whether the 'degree or strength of preference' of x_i over x_j exceeds, equals, or is less than the 'degree or strength of preference' of x_j over x_k cannot be answered by the classical preference modeling [9]. The introduction of the concept of fuzzy set as an extension of the classical concept of set when applied to a binary relation leads to the concept of a fuzzy relation. The adapted definition of a fuzzy reciprocal preference relation (FRPR) is the following one [4, 33]:

***Definition* 1 (Fuzzy Reciprocal Preference Relation (FRPR)).** A fuzzy reciprocal preference relation (FRPR) P on a finite set of alternatives $X = \{x_1, \ldots, x_n\}$ is characterised by a membership function $\mu_P \colon X \times X \longrightarrow [0,1]$, with $\mu_P(x_i, x_j) = p_{ij}$, verifying

$$\forall i, j \in \{1, \ldots, n\}:\ p_{ji} = 1 - p_{ij}. \tag{2}$$

Membership functions are subject to uncertainty arising from various sources [15, 17, 32]. Klir and Folger [29, page 12] comment:

> "... it may seem problematical, if not paradoxical, that a representation of fuzziness is made using membership grades that are themselves precise real numbers. Although this does not pose a serious problem for many applications, it is nevertheless possible to extend the concept of the fuzzy set to allow the distinction between grades of membership to become blurred."

Here Klir and Folger described blurring a fuzzy set to form an *interval-valued fuzzy set (IFS)* [14, 16, 28]:

***Definition* 2 (Interval-Valued Fuzzy Set (IFS)).** Let $INT([0,1])$ be the set of all closed subintervals of $[0,1]$ and X be an universe of discourse. An interval-valued fuzzy set (IFS) $\widetilde{A}$ on X is characterised by a membership function $\mu_{\widetilde{A}} : X \to INT([0,1])$, and it is expressed as follows:

$$A = \{(x, \mu_{\widetilde{A}}(x));\ \mu_{\widetilde{A}}(x) \in INT([0,1])\, \forall x \in X\}. \tag{3}$$

Given two interval numbers $\widetilde{a}_1 = [a_1^-, a_1^+]$ and $\widetilde{a}_2 = [a_2^-, a_2^+]$, the main interval arithmetic operations can be expressed in terms of the interval lower and upper bounds as follows [19]:

1) $\widetilde{a}_1 + \widetilde{a}_2 = [a_1^-, a_1^+] + [a_2^-, a_2^+] = [a_1^- + a_2^-, a_1^+ + a_2^+]$.

2) $\widetilde{a}_1 - \widetilde{a}_2 = [a_1^-, a_1^+] - [a_2^-, a_2^+] = [a_1^- - a_2^+, a_1^+ - a_2^-]$.

3) $\widetilde{a}_1 \cdot \widetilde{a}_2 = [a_1^-, a_1^+] \cdot [a_2^-, a_2^+] = [(a_1a_2)^-, (a_1a_2)^+]$,

$$(a_1a_2)^- = \min\{a_1^-a_2^-, a_1^-a_2^+, a_1^+a_2^-, a_1^+a_2^+\};$$

$$(a_1a_2)^+ = \max\{a_1^-a_2^-, a_1^-a_2^+, a_1^+a_2^-, a_1^+a_2^+\}.$$

4) $\widetilde{a}_1 / \widetilde{a}_2 = [a_1^-, a_1^+]/[a_2^-, a_2^+] = [(a_1/a_2)^-, (a_1/a_2)^+]$,

$$(a_1/a_2)^- = \min\{a_1^-/a_2^-, a_1^-/a_2^+, a_1^+/a_2^-, a_1^+/a_2^+\};$$

$$(a_1/a_2)^+ = \max\{a_1^-/a_2^-, a_1^-/a_2^+, a_1^+/a_2^-, a_1^+/a_2^+\},$$

provided that $0 \notin [a_2^-, a_2^+]$.

Note that real numbers $a \in \mathbb{R}$ can be represented in interval form as $[a, a]$. Two interval numbers $\widetilde{a}_1 = [a_1^-, a_1^+]$ and $\widetilde{a}_2 = [a_2^-, a_2^+]$ are equal if and only if $a_1^- = a_2^-$ and $a_1^+ = a_2^+$. An interval number $\widetilde{a} = [a^-, a^+]$ is positive when $a^- \geq 0$. The product and division of positive interval numbers can be simplified as follows:

3) $\widetilde{a}_1 \cdot \widetilde{a}_2 = [a_1^-, a_1^+] \cdot [a_2^-, a_2^+] = [a_1^-a_2^-, a_1^+a_2^+]$.

4) $\widetilde{a}_1 / \widetilde{a}_2 = [a_1^-, a_1^+]/[a_2^-, a_2^+] = [a_1^-/a_2^+, a_1^+/a_2^-]$, provided that $a_2^- > 0$.

The application of the concept of IFS to a FRPR leads to the concept of interval-valued fuzzy reciprocal preference relation (IFRPR) [22, 49]:

***Definition* 3 (Interval-Valued Fuzzy Reciprocal Preference Relation (IFRPR)).** An interval-valued fuzzy reciprocal preference relation (IFRPR) $\widetilde{P}$ on a finite set of alternatives $X = \{x_1, \ldots, x_n\}$ is characterised by a membership function $\mu_{\widetilde{P}}\colon X \times X \longrightarrow INT([0,1])$, with $\mu_{\widetilde{P}}(x_i, x_j) = \widetilde{p}_{ij} = [p_{ij}^-, p_{ij}^+]$, verifying

$$\forall i, j \in \{1, \ldots, n\}:\ \widetilde{p}_{ji} = 1 - \widetilde{p}_{ij}. \tag{4}$$

The above definition of IFRPR can be expressed in terms of the lower and upper bound of the interval-valued preference values as follows:

$$\forall i, j = 1, 2, \ldots n: \; p_{ij}^{-} + p_{ji}^{+} = p_{ij}^{+} + p_{ji}^{-} = 1. \tag{5}$$

Given two interval numbers $\widetilde{a}_1 = [a_1^-, a_1^+]$ and $\widetilde{a}_2 = [a_2^-, a_2^+]$, Xu and Da [48] proposed the following possibility degree (PD) to measure the degree up to which the ordering relation $\widetilde{a}_1 \succ \widetilde{a}_2$ holds:

$$P(\tilde{a}_1 \succ \tilde{a}_2) = \max\left\{1 - \max\left\{\frac{a_2^+ - a_1^-}{a_1^+ - a_1^- + a_2^+ - a_2^-}, 0\right\}, 0\right\} \tag{6}$$

2.1. Consistency of IFRPR

Consistency of FRPRs is based on the notion of transitivity, in the sense that if alternative x_i is preferred to alternative x_j ($p_{ij} \geq 0.5$) and this one to x_k ($p_{jk} \geq 0.5$), then alternative x_i should be preferred to x_k ($p_{ik} \geq 0.5$). This transitivity notion is normally referred to as *weak stochastic transitivity* [31]. Later, Tanino [38] introduced the concept of multiplicative transitivity of FRPRs as follows:

***Definition* 4 (Multiplicative Transitive FRPR).** A FRPR $P = (p_{ij})$ on a finite set of alternatives X is multiplicative transitive if and only if

$$\frac{p_{ji}}{p_{ij}} = \frac{p_{jk}}{p_{kj}} \cdot \frac{p_{ki}}{p_{ik}} \quad \forall i, k, j \in \{1, 2, \ldots n\} \tag{7}$$

is verified by non zero preference values.

Obviously, multiplicative transitivity property extends weak stochastic transitivity, and therefore extends the classical transitivity property of crisp preference relations. Furthermore, Chiclana et al. [9] proved that

$$p_{ij} \cdot p_{jk} \cdot p_{ki} = p_{ik} \cdot p_{kj} \cdot p_{ji} \quad \forall i, k, j$$

is equivalent to

$$p_{ij} \cdot p_{jk} \cdot p_{ki} = p_{ik} \cdot p_{kj} \cdot p_{ji} \quad \forall i < j < k,$$

and, ultimately, they characterised the formulation of the cardinal consistency of FRPRs via representable uninorms. Because the cardinal consistency with the conjunctive representable cross ratio uninorm is equivalent to Tanino's multiplicative transitivity property, and any two representable uninorms are order-isomorphic, it was proved that multiplicative transitivity is the most appropriate property to model consistency of FRPRs. This is captured in the following definition [9]:

***Definition* 5 (Consistent FRPR).** A FRPR $P = (p_{ij})$ on a finite set of alternatives X is consistent if and only if

$$U(p_{ik}, p_{kj}) = \begin{cases} 0 & (p_{ik}, p_{kj}) \in \{(0,1), (1,0)\} \\ \dfrac{p_{ik} \cdot p_{kj}}{p_{ik} \cdot p_{kj} + (1 - p_{ik}) \cdot (1 - p_{kj})} & otherwise \end{cases} \tag{8}$$

Because FRPRs are particular types of IFRPRs, we can extend the notion of multiplicative transitivity of FRPRs to the case of IFRPRs as per the following definition:

***Definition* 6 (Multiplicative Transitive IFRPR).** An IFRPR $\widetilde{P} = (\widetilde{p}_{ij})_{n\times n}$ on a finite set of alternatives X is multiplicative transitive if and only if

$$\frac{\widetilde{p}_{ji}}{\widetilde{p}_{ij}} = \frac{\widetilde{p}_{ki}}{\widetilde{p}_{ik}} \cdot \frac{\widetilde{p}_{jk}}{\widetilde{p}_{kj}}, \quad i < k < j \tag{9}$$

Notice that reciprocity of preferences and division of interval number yield:

$$\frac{\widetilde{p}_{ji}}{\widetilde{p}_{ij}} = \frac{[p_{ji}^-, p_{ji}^+]}{[p_{ij}^-, p_{ij}^+]} = \left[\frac{p_{ji}^-}{p_{ij}^+}, \frac{p_{ji}^+}{p_{ij}^-}\right] = \left[\frac{1-p_{ij}^+}{p_{ij}^+}, \frac{1-p_{ij}^-}{p_{ij}^-}\right] = \left[\frac{1}{p_{ij}^+} - 1, \frac{1}{p_{ij}^-} - 1\right].$$

Applying the product of positive interval numbers and the equality of interval numbers, we have:

$$\frac{1}{p_{ij}^+} - 1 = \left(\frac{1}{p_{ik}^+} - 1\right) \cdot \left(\frac{1}{p_{kj}^+} - 1\right);$$

$$\frac{1}{p_{ij}^-} - 1 = \left(\frac{1}{p_{ik}^-} - 1\right) \cdot \left(\frac{1}{p_{kj}^-} - 1\right).$$

The above expressions can be rewritten as follows:

$$p_{ij}^- = \frac{p_{ik}^- \cdot p_{kj}^-}{p_{ik}^- \cdot p_{kj}^- + (1-p_{ik}^-)\cdot(1-p_{kj}^-)};$$

$$p_{ij}^+ = \frac{p_{ik}^+ \cdot p_{kj}^+}{p_{ik}^+ \cdot p_{kj}^+ + (1-p_{ik}^+)\cdot(1-p_{kj}^+)}.$$

Finally, because function $f(x) = x/(x+a)$ is monotone increasing with respect to the variable x when $a > 0$, then it is clear that

$$0 \le p_{ij}^- \le p_{ij}^+ \le 1 \quad \forall i, j.$$

Therefore, we have proved the following result:

***Theorem* 1.** *If an IFRPR $\widetilde{P}$ is multiplicative transitive, then we have*

1) $p_{ij}^- = \dfrac{p_{ik}^- \cdot p_{kj}^-}{p_{ik}^- \cdot p_{kj}^- + (1-p_{ik}^-)\cdot(1-p_{kj}^-)}, \quad i < k < j$

2) $p_{ij}^+ = \dfrac{p_{ik}^+ \cdot p_{kj}^+}{p_{ik}^+ \cdot p_{kj}^+ + (1-p_{ik}^+)\cdot(1-p_{kj}^+)}, \quad i < k < j$

The following definition is therefore justified:

***Definition* 7 (Consistent IFRPR).** An IFRPR $\widetilde{P} = (\widetilde{p}_{ij})_{n\times n}$ on a finite set of alternatives X is consistent if and only if

$$\widetilde{U}(\widetilde{p}_{ik}, \widetilde{p}_{kj}) = \begin{cases} 0, & (\widetilde{p}_{ik}, \widetilde{p}_{kj}) \in \{(0,1),(1,0)\} \\ \left[\dfrac{p_{ik}^- \cdot p_{kj}^-}{p_{ik}^- \cdot p_{kj}^- + (1-p_{ik}^-)\cdot(1-p_{kj}^-)}, \dfrac{p_{ik}^+ \cdot p_{kj}^+}{p_{ik}^+ \cdot p_{kj}^+ + (1-p_{ik}^+)\cdot(1-p_{kj}^+)}\right] & , otherwise \end{cases} \tag{10}$$

The consistency property (10) can be used to compute consistency based estimated values of the elements of a given IFRPR. Indeed, given an IFRPR $\widetilde{P} = (\widetilde{p_{ij}})$, the interval-valued preference value $\widetilde{p_{ij}}$ can be partially $\widetilde{U}$-estimated using an intermediate alternative x_k $(i < k < j)$ as follows:

$$\widetilde{up}_{ij}^{k} = \widetilde{U}(\widetilde{p}_{ik}, \widetilde{p}_{kj}). \tag{11}$$

Then, the global consistency based estimated value can be computed as the average of the partially $\widetilde{U}$-estimated values obtained using all possible intermediate alternatives:

$$up_{ij}^{-} = \frac{\sum_{k=i+1}^{j-1} up_{ij}^{k-}}{j-i-1}; \quad up_{ij}^{+} = \frac{\sum_{k=i+1}^{j-1} up_{ij}^{k+}}{j-i-1}.$$

Therefore, given an IFRPR, $\widetilde{P} = (\widetilde{p_{ij}})$, the following $\widetilde{U}$-consistency estimated IFRPR, $\widetilde{UP} = (\widetilde{up_{ij}})_{n\times n} = \left(\left[up_{ij}^{-}, up_{ij}^{+}\right]\right)_{n\times n}$ can be constructed:

$$up_{ij}^{-} = \begin{cases} p_{ij}^{-}, & i \leq j \leq i+1 \\ \dfrac{\sum_{k=i+1}^{j-1} up_{ij}^{k-}}{j-i-1}, & i+1 \leq k \leq j \\ 1 - up_{ji}^{+}, & i > j \end{cases} \tag{12}$$

and

$$up_{ij}^{+} = \begin{cases} p_{ij}^{+}, & i \leq j \leq i+1 \\ \dfrac{\sum_{k=i+1}^{j-1} up_{ij}^{k+}}{j-i-1}, & i+1 \leq k \leq j \\ 1 - up_{ji}^{-}, & i > j \end{cases} \tag{13}$$

Because the $\widetilde{U}$-consistency property is the only consistency property that is used in the rest of the paper, the symbol $\widetilde{U}$ will not be used unless it is necessary to differentiate between different consistency properties.

2.2. Consistency Indexes of IFRPRs

If the information provided in an IFRPR is completely consistent, then it is $\widetilde{p}_{ij} = \widetilde{up}_{ij}$. However, in real decision making problems experts are not always fully consistent. As a result, it is necessary to measure their degree of inconsistency. The distance between the values $\widetilde{p}_{ij}$ and $\widetilde{up}_{ij}$, $d(\widetilde{p}_{ij}, \widetilde{up}_{ij})$, can be used in measuring the level of consistency of an IFRPR at its three different levels: pair of alternatives, alternatives and relation.

***Definition* 8 (Pair of Alternatives Consistency Index (CI_{ij})).** Let $\widetilde{P}$ be an IFRPR and $\widetilde{UP}$ its corresponding consistency estimated IFRPR. The consistent index at the pair of alternatives (x_i, x_j), CI_{ij}, is:

$$CI_{ij} = 1 - d(\widetilde{p}_{ij}, \widetilde{up}_{ij}). \tag{14}$$

The higher the value of CI_{ij}, the more consistent is $\widetilde{p}_{ij}$ with respect to the rest of preference values involving alternatives x_i and x_j. Notice that $CI_{ij} = CI_{ji}$.

The consistency index at the level of alternatives is obtained by aggregating all the consistency index values of its corresponding pair of alternatives:

***Definition* 9 (Alternative Consistency Index (CI_i)).** The consistency index associated to a particular alternative x_i is

$$CI_i = \frac{\sum_{\substack{j=1 \\ i \neq j}}^{n} CI_{ij}}{(n-1)} \tag{15}$$

When $CI_i = 1$ then all the preference values involving alternative x_i are fully consistent.

The global consistency index of an IFRPR is defined as the aggregated value of all individual alternative consistency indexes:

***Definition* 10 (IFRPR Consistency Index (CI)).** The consistency index of an IFRPR $\widetilde{P}$ is defined as follows:

$$CI = \frac{\sum_{i=1}^{n} CI_i}{n} \tag{16}$$

Notice that $CI = 1$ if and if only if $\sum_{i,j=1, i\neq j}^{n} CI_{ij} = n \cdot (n-1)$. Because $CI_{ij} \in [0,1]$, then we have that $\sum_{i,j=1, i\neq j}^{n} CI_{ij} = n \cdot (n-1)$ if and if only if $CI_{ij} = 1 \forall i \neq j$ and therefore we have that $d(\widetilde{p}_{ij}, \widetilde{up}_{ij}) = 0 (\forall i \neq j)$, which means that the IFRPR, $\widetilde{P}$, and its corresponding consistency based estimated IFRPR, $\widetilde{UP}$, coincide. Therefore, we have proved the following result:

***Proposition* 1.** *An IFRPR $\widetilde{P}$ is consistent if and if only if $CI = 1$.*

Example 1. Computation of Consistency Indexes. Suppose four different experts $\{e_1, e_2, e_3, e_4\}$ provide the following IFRPRs over a set of four alternatives $\{x_1, x_2, x_3, x_4\}$:

$$\widetilde{P}^1 = \begin{pmatrix} - & [0.3, 0.5] & [0.4, 0.6] & [0.5, 0.7] \\ [0.5, 0.7] & - & [0.5, 0.8] & [0.5, 0.6] \\ [0.4, 0.6] & [0.2, 0.5] & - & [0.4, 0.6] \\ [0.3, 0.5] & [0.4, 0.5] & [0.4, 0.6] & - \end{pmatrix} \quad \widetilde{P}^2 = \begin{pmatrix} - & [0.4, 0.5] & [0.3, 0.4] & [0.4, 0.6] \\ [0.5, 0.6] & - & [0.5, 0.6] & [0.6, 0.7] \\ [0.6, 0.7] & [0.4, 0.5] & - & [0.4, 0.5] \\ [0.4, 0.6] & [0.3, 0.4] & [0.5, 0.6] & - \end{pmatrix}$$

$$\widetilde{P}^3 = \begin{pmatrix} - & [0.3, 0.6] & [0.4, 0.5] & [0.2, 0.3] \\ [0.4, 0.7] & - & [0.3, 0.5] & [0.3, 0.4] \\ [0.5, 0.6] & [0.5, 0.7] & - & [0.6, 0.7] \\ [0.7, 0.8] & [0.6, 0.7] & [0.3, 0.4] & - \end{pmatrix} \quad \widetilde{P}^4 = \begin{pmatrix} - & [0.4, 0.6] & [0.5, 0.8] & [0.5, 0.8] \\ [0.4, 0.6] & - & [0.4, 0.5] & [0.5, 0.7] \\ [0.2, 0.5] & [0.5, 0.6] & - & [0.4, 0.5] \\ [0.2, 0.5] & [0.3, 0.5] & [0.5, 0.6] & - \end{pmatrix}$$

The consistency based estimated IFRPRs are:

$$\widetilde{UP}^1 = \begin{pmatrix} - & [0.30, 0.50] & [0.30, 0.80] & [0.30, 0.65] \\ [0.50, 0.70] & - & [0.50, 0.80] & [0.40, 0.86] \\ [0.20, 0.70] & [0.20, 0.50] & - & [0.40, 0.60] \\ [0.35, 0.70] & [0.14, 0.60] & [0.40, 0.60] & - \end{pmatrix}$$

$$\widetilde{UP}^2 = \begin{pmatrix} - & [0.40, 0.50] & [0.40, 0.60] & [0.36, 0.55] \\ [0.50, 0.60] & - & [0.50, 0.60] & [0.40, 0.60] \\ [0.40, 0.60] & [0.40, 0.50] & - & [0.40, 0.50] \\ [0.45, 0.64] & [0.40, 0.60] & [0.50, 0.60] & - \end{pmatrix}$$

$$\widetilde{UP}^3 = \begin{pmatrix} - & [0.30, 0.60] & [0.16, 0.60] & [0.33, 0.60] \\ [0.40, 0.70] & - & [0.30, 0.50] & [0.39, 0.70] \\ [0.40, 0.84] & [0.50, 0.70] & - & [0.60, 0.70] \\ [0.40, 0.67] & [0.30, 0.61] & [0.30, 0.40] & - \end{pmatrix}$$

$$\widetilde{UP}^4 = \begin{pmatrix} - & [0.40, 0.60] & [0.31, 0.60] & [0.40, 0.79] \\ [0.40, 0.60] & - & [0.40, 0.50] & [0.31, 0.50] \\ [0.40, 0.69] & [0.50, 0.60] & - & [0.40, 0.50] \\ [0.21, 0.60] & [0.50, 0.69] & [0.50, 0.60] & - \end{pmatrix}$$

Using the Hamming distance [10, 49],

$$d(\widetilde{p}_{ij}, \widetilde{up}_{ij}) = \frac{1}{2}\Big(|p_{ij}^- - up_{ij}^-| + |p_{ij}^+ - up_{ij}^+|\Big).$$

we have:

- The pair of alternatives level consistency indexes are:

$$(CI_{ij}^1) = \begin{pmatrix} - & 1.000 & 0.850 & 0.875 \\ 1.000 & - & 1.000 & 0.820 \\ 0.850 & 1.000 & - & 1.000 \\ 0.875 & 0.820 & 1.000 & - \end{pmatrix} \quad (CI_{ij}^2) = \begin{pmatrix} - & 1.0000 & 0.850 & 0.955 \\ 1.000 & - & 1.000 & 0.850 \\ 0.850 & 1.000 & - & 1.000 \\ 0.955 & 0.850 & 1.000 & - \end{pmatrix}$$

$$(CI_{ij}^3) = \begin{pmatrix} - & 1.000 & 0.830 & 0.785 \\ 1.000 & - & 1.000 & 0.805 \\ 0.830 & 1.000 & - & 1.000 \\ 0.785 & 0.805 & 1.000 & - \end{pmatrix} \quad (CI_{ij}^4) = \begin{pmatrix} - & 1.000 & 0.805 & 0.945 \\ 1.000 & - & 1.000 & 0.805 \\ 0.805 & 1.000 & - & 1.000 \\ 0.945 & 0.805 & 1.000 & - \end{pmatrix}$$

- The alternatives level consistency indexes are:

$$(CI_i^1) = (0.908, 0.940, 0.950, 0.898); \quad (CI_i^2) = (0.935, 0.950, 0.950, 0.935);$$
$$(CI_i^3) = (0.872, 0.935, 0.943, 0.863); \quad (CI_i^4) = (0.917, 0.935, 0.935, 0.917).$$

- The experts' consistency indexes are: $CI^1 = 0.924, CI^2 = 0.943, CI^3 = 0.903, CI^4 = 0.926.$

If the threshold value of consistency is set at $\lambda = 0.9$, then all IFRPRs can be considered as acceptable in terms of consistency.

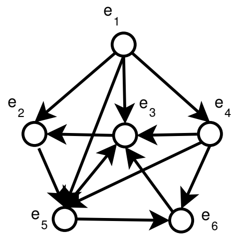
Sociometric	Graph	Algebraic
$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$		e_1Re_2 e_4Re_3 e_1Re_3 e_4Re_5 e_1Re_4 e_4Re_6 e_1Re_5 e_5Re_3 e_2Re_5 e_5Re_6 e_3Re_2 e_6Re_3

Table 1: Different representation schemes in Social Network Analysis

3. Trust degree based on Social Network Analysis

Before aggregating individual IFRPRs to derive a collective IFRPR, it is necessarily to determine the weights associated to each one of the experts. In most GDM problems, the weights of experts assumed to be known beforehand. However, this might not be a realistic assumption, and therefore it is worth and interesting to provide alternative ways to obtain that information. Trust between experts in a group plays an important role in any GDM process, and therefore it should also be taken into account as a reliable source to be used in deriving aggregation weights for individual experts. An approach to make this possible using the concepts of Social Network Analysis (SNA) to model and measure the level/degree of trust of an expert within a group is given below.

SNA studies the relationships between social entities like members of a group, corporations or nations [18, 37, 40]. Therefore, it enables us to examine the structural and locational properties including centrality, prestige, structural balance and trust relationship, among others. Consequently, a SNA based methodology could be appropriate to model the concept of trust degree (TD) as well as to make possible its measurement to reflect the actual trust relationships between experts in a group.

The main three elements in SNA analysis are: the set of actors, the relations themselves, and the actor attributes (see Table 1). We can refer to important network concepts in a unify manner, using the three different and possible representation schemes:

- Sociometric: relational data are often presented in two-ways matrices called sociomatrix.
- Graph theoretic: the network is viewed as a graph consisting of nodes joined by lines.
- Algebraic: allows to distinguish several distinct relations and represent combinations of relations.

Notice that the sociomatrix provided in the above example is a binary or crisp relation. However, in many situations, it may not be suitable to represent the relation in a crisp way because this is not clear cut defined as described in Section 2. In the following, we assume that the relation of trustworthiness between experts is interval-valued:

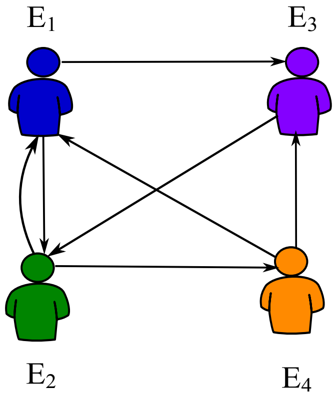


Figure 1: Graph representation of the sociomatrix

***Definition* 11.** An interval-valued fuzzy sociomatrix S_L on E is a relation in $E \times E$ with membership function $\mu_{S_L} : E \times E \to INT([0,1])$, $\mu_{S_L}(e_k, e_h) = S_{kh}$.

Example 2. Suppose that the four different experts $\{e_1, e_2, e_3, e_4\}$ trust relationship can be represented in (directed) graph form as in Figure 1 with the following interval-valued fuzzy sociomatrix S_L:

$$S_L = \begin{pmatrix} - & [0.6, 0.7] & [0.8, 0.9] & - \\ [0.3, 0.4] & - & - & [0.7, 0.9] \\ - & [0.2, 0.4] & - & - \\ [0.6, 0.8] & - & [0.2, 0.4] & - \end{pmatrix}$$

3.1. Building Trust Relationship

Given a directed graph, the in-degree of centrality can be used to measure the importance of nodes in the network:

***Definition* 12.** Let $G = (E, L, \omega)$ be a directed graph, $E = \{e_1, \ldots, e_m\}$ be the set of nodes and $L = \{l_1, \ldots, l_q\}$ be the set of directed lines, or arcs, between pairs of nodes and the set of interval assessments $\omega = \{\omega_1^L, \ldots, \omega_q^L\}$ attached to the lines (or arcs), $S_L = (S_{kh})_{m \times m}$ be the sociomatrix associated with the graph $G = (E, L, \omega)$, then the relative node in-degree centrality index obtained from the sociomatrix is computed as follows:

$$C_D^L(e_h) = \frac{1}{m-1} \sum_{k=1}^{m} S_{kh} \tag{17}$$

Example 3. ***(Example 2 continuation. Trust in-degree centrality indexes)*** According to expression (17), we obtain the following trust in-degree centrality indexes:

$$C_D^L(e_1) = [0.30, 0.40]; C_D^L(e_2) = [0.27, 0.37];$$

$$C_D^L(e_3) = [0.33, 0.43]; C_D^L(e_4) = [0.23, 0.30].$$

3.2. Computing Trust Degrees

Based on the in-degree centrality index, we define the trust degree of each expert in the group as follows:

***Definition* 13 (Trust Degree (TD)).** Let $G = (E, L, \omega)$ be a directed graph representing the trust relationship between the group of experts $E = \{e_1, \ldots, e_m\}$ and $\{C_D^L(e_1), \ldots, C_D^L(e_h)\}$ be the set of in-degree centrality indexes. The trust degree (TD) of expert e_h can be expressed as:

$$TD^h = \frac{1}{m-1} \sum_{k=1}^{m} P(C_D^L(e_h) \succ C_D^L(e_k)) \tag{18}$$

where $P(C_D^L(e_h) \succ C_D^L(e_k))$ is possibility degree of $C_D^L(e_h) \geq C_D^L(e_k)$ as per expression (6).

Example 4. ***(Example 2 continuation. Trust degrees)*** According to expression (18), we obtain the following trust degrees of experts:

$$TD^1 = 0.333;\ TD^2 = 0.228;\ TD^3 = 0.242;\ TD^4 = 0.197.$$

4. Trust-Consensus Model with IFRPRs

In GDM problems, the selection process involves two main steps: *aggregation* of individual preferences and *exploitation* of the collective preference [21, 27]. In order to aggregate the individual preference relations into a collective one, it is important to determine the weights associated to each expert. As mentioned before in Section 3, trust between experts in a group should be taken into account in deriving aggregation weights for individual experts, and SNA allows for this to be realised via the TD concept as per Definition 13, which can be considered as a subjective reliable information source of the experts' weights. Additionally, CL can be considered as an objective reliable information source of the experts' weights as it is derived from their own opinions on the decision problem to solve. Thus, it is proposed the combination of both reliable sources, trust and consensus, to develop a trust-consensus based approach to compute the weights of experts.

Clearly, it is preferable that the group of experts achieve a high consensus level among their preferences before applying the selection process to derive the final solution of the decision making problem. The consensus process involves three issues: the analysis of intra-consistency, i.e. the modelling and measurement of each individual expert's consistency state; the study of inter-consistency, i.e. the modelling and computation of the group agreement degree; and the development of a feedback mechanism.

1. Intra-consistency refers to the existence of self contradiction in the opinions provided by experts individually and its measurement. Obviously, consistent information, i.e., information which

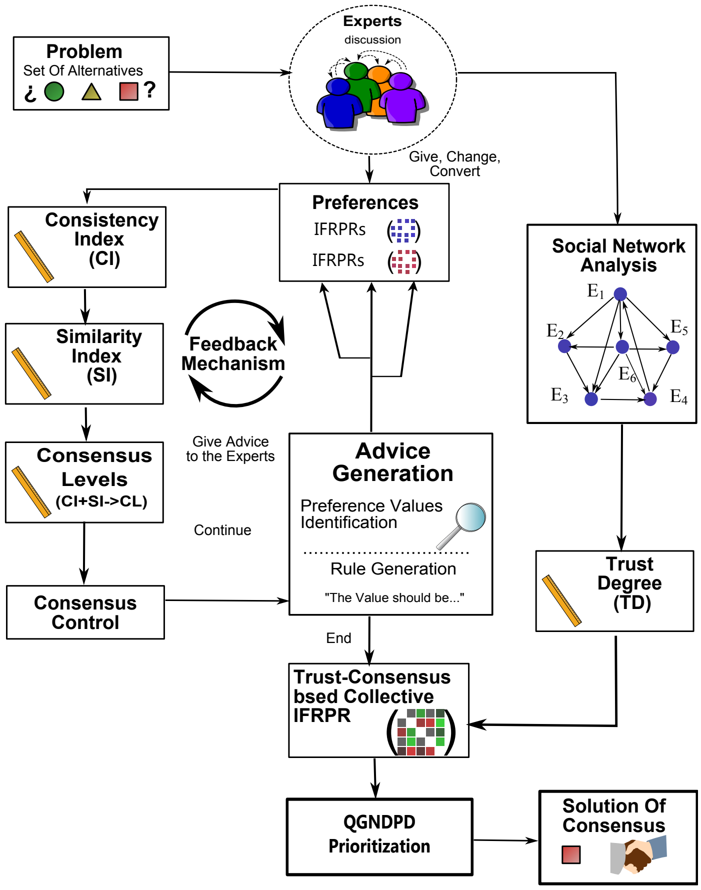


Figure 2: Trust-Consensus based process for GDM with IFRPRs

does not imply any kind of contradiction, is more relevant or important than information containing some contradictions. In the case of information being represented using IFRPRs, this issue was the focus of Section 2, and in particular CIs for IFRPRs were developed in Section 2.2.

2. Inter-consistency refers to the measurement of the group agreement, i.e. how close or similar are the experts' opinions on the problem to solve. Similarity indexes (SIs) are needed to measure the actual level of group agreement in the decision problem. The joint implementation of CIs and SIs are proposed to measure the group consensus level (CL).

3. If the consensus level reaches a threshold value, agreed by the group experts before hand, then the GDM selection process is carried out; otherwise a feedback mechanism generating personalised recommendations to experts is activated. These recommendations will not only tell the experts the preference values they should change, but also include the values the experts should use, to increase the level of agreement in a consistent way.

The trust-consensus based process for GDM with IFRPRs is depicted in Figure 2. Specifically, the consensus process consists of the following four steps: (1) Computing the SNA based TDs; (2) Determining SIs; (3) Calculating CLs; and (4) Developing the feedback mechanism.The first step has already been covered in Section 3. The remaining steps will be presented in more detail in the following subsections. The aggregation and prioritisation operations of the selection process are the focus of Section 5. A step-by-step example to illustrate the computation processes involved in each step of the consensus-selection process is also provided.

4.1. Similarity Indexes of IFRPRs

Let $\widetilde{P}^h = (\widetilde{p}_{ij}^h)_{n\times n}$ and $\widetilde{P}^l = (\widetilde{p}_{ij}^l)_{n\times n}$ be the IFRPRs provided by experts e_h and e_l, respectively. A distance function (d) between interval numbers is used to define the similarity indexes between these experts as follows:

***Definition* 14 (Pair of Alternatives Similarity Index).** The similarity index $SI(\widetilde{p}_{ij}^h, \widetilde{p}_{ij}^l)$ between experts e_h and e_l on the pair of alternatives (x_i, x_j) is:

$$SI(\widetilde{p}_{ij}^h, \widetilde{p}_{ij}^l) = 1 - d(\widetilde{p}_{ij}^h, \widetilde{p}_{ij}^l) \tag{19}$$

When $SI(\widetilde{p}_{ij}^h, \widetilde{p}_{ij}^l) = 1$, we have that $d(\widetilde{p}_{ij}^h, \widetilde{p}_{ij}^l)$=0 and $\widetilde{p}_{ij}^h$ is equal to $\widetilde{p}_{ij}^l$. Therefore, the higher the value $SI(\widetilde{p}_{ij}^h, \widetilde{p}_{ij}^l)$, the more similar are $\widetilde{p}_{ij}^h$ and $\widetilde{p}_{ij}^l$.

Similarity indexes between experts can also be defined at the level of the alternatives and at the level of the relation:

***Definition* 15 (Alternatives Similarity Index).** The similarity index $SI(\widetilde{p}_i^h, \widetilde{p}_i^l)$ between experts e_h and e_l on the alternative x_i is:

$$SI(\widetilde{p}_i^h, \widetilde{p}_i^l) = \frac{\sum_{j=1}^n SI(\tilde{p}_{ij}^h, \tilde{p}_{ij}^l)}{n}. \tag{20}$$

If $SI(\widetilde{p}_i^h, \widetilde{p}_i^l) = 1$, then all the preference values involving the alternative x_i for both experts are the same. Thus, the higher the value $SI(\widetilde{p}_i^h, \widetilde{p}_i^l)$, the closer are these experts in their preference for the alternative x_i.

***Definition* 16 (IFRPR Similarity Index (SI)).** The similarity degree $SI(\widetilde{P}^h, \widetilde{P}^l)$ between experts e_h and e_l on the relation, and therefore on the whole set of alternatives, is:

$$\begin{aligned} SI(\widetilde{P}^h, \widetilde{P}^l) &= \frac{\sum_{i=1}^n SI(\tilde{p}_i^h, \tilde{p}_i^l)}{n} \\ &= \frac{\sum_{i=1}^n \sum_{j=1}^n SI(\tilde{p}_{ij}^h, \tilde{p}_{ij}^l)}{n^2} \end{aligned} \tag{21}$$

When $SI(\widetilde{P}^h, \widetilde{P}^l) = 1$, then it is $\widetilde{P}^h = \widetilde{P}^l$. Consequently, the higher the value $SI(\widetilde{P}^h, \widetilde{P}^l)$, the closer are the experts E_h and E_l on their preferences on decision problem to solve.

The similarity index SI verifies the following properties:

***Proposition* 2.** *Let $\widetilde{P}^1$, $\widetilde{P}^2$ and $\widetilde{P}^3$ be three IFRPRs, then we have*

1) *Reflexivity: $SI(\widetilde{P}^1, \widetilde{P}^1)$=1,*

2) *Symmetry:* $SI(\widetilde{P}^1, \widetilde{P}^2) = SI(\widetilde{P}^2, \widetilde{P}^1)$,

3) *Transitivity: If* $SI(\widetilde{P}^1, \widetilde{P}^2)$*=1 and* $SI(\widetilde{P}^2, \widetilde{P}^3)$*=1, then* $SI(\widetilde{P}^1, \widetilde{P}^3)$*=1,*

The similarity index of an expert with the rest of the group of experts at the three different levels of a relation are defined next:

Level 1. *Similarity index on the pairs.* The similarity index of an expert e_h on the pair (x_i, x_j) to the rest of experts is calculated as:

$$SI_{ij}^h = \frac{\sum_{l=1, l\neq h}^{m} SI(\tilde{p}_{ij}^h, \tilde{p}_{ij}^l)}{m-1} \tag{22}$$

Level 2. *Similarity index on the alternatives.* The similarity index of an expert e_h on the alternative x_i to the rest of experts is calculated as:

$$SI_i^h = \frac{\sum_{j=1}^{n} SI_{ij}^h}{n} \tag{23}$$

Level 3. *Similarity index on the relation.* The similarity index of an expert e_h on his/her preference relation to the rest of experts is calculated as:

$$SI^h = \frac{\sum_{i=1}^{n} SI_i^h}{n} \tag{24}$$

Example 5. ***(Example 1 continuation).*** **Computation of Similarity Indexes.** Using again the Hamming distance we have:

I) The similarity indexes on the pair of alternatives for each expert are:

$$(SI_{ij}^1) = \begin{pmatrix} - & 0.933 & 0.883 & 0.833 \\ 0.933 & - & 0.817 & 0.883 \\ 0.883 & 0.817 & - & 0.917 \\ 0.833 & 0.883 & 0.917 & - \end{pmatrix} \quad (SI_{ij}^2) = \begin{pmatrix} - & 0.933 & 0.817 & 0.833 \\ 0.933 & - & 0.883 & 0.850 \\ 0.817 & 0.883 & - & 0.917 \\ 0.833 & 0.850 & 0.917 & - \end{pmatrix}$$

$$(SI_{ij}^3) = \begin{pmatrix} - & 0.933 & 0.883 & 0.667 \\ 0.933 & - & 0.850 & 0.750 \\ 0.883 & 0.850 & - & 0.817 \\ 0.667 & 0.750 & 0.817 & - \end{pmatrix} \quad (SI_{ij}^4) = \begin{pmatrix} - & 0.933 & 0.783 & 0.800 \\ 0.933 & - & 0.883 & 0.883 \\ 0.783 & 0.883 & - & 0.917 \\ 0.800 & 0.883 & 0.917 & - \end{pmatrix}$$

II) The similarity indexes on alternatives for each expert are:

$$(SI_i^1) = (0.883, 0.878, 0.872, 0.878); \quad (SI_i^2) = (0.861, 0.889, 0.872, 0.867);$$
$$(SI_i^3) = (0.828, 0.844, 0.850, 0.744); \quad (SI_i^4) = (0.839, 0.900, 0.861, 0.867).$$

III) The similarity indexes on the set of alternatives for each expert are:

$$SI^1 = 0.878, SI^2 = 0.872, SI^3 = 0.817, SI^4 = 0.864.$$

4.2. Calculating Consensus Levels of IFRPRs

CLs are defined as a linear combination of CIs and SIs. When CL reaches a threshold value, agreed by the group of experts, the selection process is carried out; otherwise a feedback mechanism is activated, and personalised recommendations are generated to support the individual experts. The feedback process ceases when the consensus threshold level is achieved. The feedback recommendations will help the experts to identify the preference values that should be considered for changing.

Experts' CLs are computed as follows:

$$\forall h: \quad CL^h = \psi \cdot CI^h + (1-\psi) \cdot SI^h \tag{25}$$

with $\psi \in [0,1]$ a parameter to control the weights of both consistency and similarity criteria. When all CL^h are above a minimum satisfaction threshold value $\gamma \in [0.5,1]$, the consensus reaching process finishes and the selection process is applied.

Notice that consensus is defined as the full and unanimous agreement of all the experts regarding all the feasible alternatives. This, however, is inconvenient because it only allows differentiating between two states, namely, the existence and absence of consensus. Also, the chances for reaching such a full agreement are rather low. Therefore, the threshold value $\gamma < 1$. Also in most cases, if more than half of people achieve consensus, the decision-making result may be acceptable. Thus, $\gamma \geq 0.5$. Consequently, we can assume that the threshold value $\gamma \in [0.5,1)$.

Example 6. ***(Example 1 continuation).*** **Computation of Consensus Level.**
Let $\psi = 0.5$ and fix the minimum threshold value $\gamma = 0.9$. The consensus levels for each one of the expert are:

$$CL^1 = 0.906; \quad CL^2 = 0.914; \quad CL^3 = 0.869; \quad CL^4 = 0.901.$$

Since $CL^3 < 0.9$, then expert e_3 will receive feedback advice on how to change his/her preferences to achieve a higher consensus level.

4.3. Feedback Mechanism

The feedback mechanism consists of two steps: *Identification of the interval-valued fuzzy preference values* that should be changed and *Generation of advice*. Both stages are described in detail below:

(1) *Identification of the Interval-Valued Fuzzy Preference Values*: The set of interval-valued fuzzy preference values that contribute less to reach an acceptable consensus level is identified as follows:

Step 1. The set of experts with consensus levels below the threshold value γ is identified:

$$EXPCH = \{h \mid CL^h < \gamma\} \tag{26}$$

Step 2. For experts identified in step 1, we identify those alternatives with a consensus level below γ:

$$ALT = \{(h,i) \mid h \in EXPCH \ \wedge \psi \cdot CI_i^h + (1-\psi) \cdot SI_i^h < \gamma\} \tag{27}$$

Step 3. Finally, we identify the interval-valued fuzzy preference values for the experts and alternatives identified in steps 1 and 2 that need to be changed:

$$APS = \{(h,i,j) \mid (h,i) \in ALT \ \wedge \psi \cdot CI_{ij}^h + (1-\psi) \cdot SI_{ij}^h < \gamma\} \tag{28}$$

Example 7. ***(Example 1 continuation).*** **Interval-Valued Fuzzy Preference Values to Change.** The set of 3-tuples APS identified as contributing less to consensus are:

$$APS = \{(3,1,3),(3,1,4),(3,2,4),(3,3,1),(3,4,1),(3,4,2)\}$$

(2) *Generation of Advice*: The feedback mechanism generates personalised recommendations rules to the experts and for the preference values previously identified in APS containing the new preference values to use in order to reach a higher consensus state.

For all $(h,i,k) \in APS$, the following rule is feed-backed to the corresponding expert:

"Change your preference value for the pair of alternatives (i,j) to a value closer to $\overline{\overline{p}}_{ij}^h$:"

$$\overline{\overline{p}}_{ij}^h = \psi \cdot \widetilde{p}_{ij}^h + (1-\psi) \cdot \overline{p}_{ij}^h, \tag{29}$$

where $\overline{p}_{ij}^h = (\sum_{l=1,l\neq h}^m \widetilde{p}_{ij}^l)/(m-1)$.

Example 8. ***(Example 1 continuation).*** **Generation of Advice.** The recommendations for expert e_3 are:

- Change your preference value for the pair of alternatives (1,3) to a value closer to [0.40,0.54].
- Change your preference value for the pair of alternatives (1,4) to a value closer to [0.31,0.46].
- Change your preference value for the pair of alternatives (2,4) to a value closer to [0.39,0.51].
- Change your preference value for the pair of alternatives (3,1) to a value closer to [0.46,0.60].
- Change your preference value for the pair of alternatives (4,1) to a value closer to [0.54,0.69].
- Change your preference value for the pair of alternatives (4,2) to a value closer to [0.49,0.61].

After expert e_3 implements the new recommended interval-valued fuzzy preference values, a new round of the consensus process is carried out.

Example 9. ***(Example 1 continuation).*** **Second consensus round.** The new IFRPRs for the second round of the consensus process are:

$$\widetilde{P}^1 = \begin{pmatrix} - & [0.3, 0.5] & [0.4, 0.6] & [0.5, 0.7] \\ [0.5, 0.7] & - & [0.5, 0.8] & [0.5, 0.6] \\ [0.4, 0.6] & [0.2, 0.5] & - & [0.4, 0.6] \\ [0.3, 0.5] & [0.4, 0.5] & [0.4, 0.6] & - \end{pmatrix} \quad \widetilde{P}^2 = \begin{pmatrix} - & [0.4, 0.5] & [0.3, 0.4] & [0.4, 0.6] \\ [0.5, 0.6] & - & [0.5, 0.6] & [0.6, 0.7] \\ [0.6, 0.7] & [0.4, 0.5] & - & [0.4, 0.5] \\ [0.4, 0.6] & [0.3, 0.4] & [0.5, 0.6] & - \end{pmatrix}$$

$$\widetilde{P}^3 = \begin{pmatrix} - & [0.30, 0.60] & [0.40, 0.54] & [0.31, 0.46] \\ [0.40, 0.70] & - & [0.30, 0.50] & [0.39, 0.51] \\ [0.46, 0.60] & [0.50, 0.70] & - & [0.60, 0.70] \\ [0.54, 0.69] & [0.49, 0.61] & [0.30, 0.40] & - \end{pmatrix} \quad \widetilde{P}^4 = \begin{pmatrix} - & [0.4, 0.6] & [0.5, 0.8] & [0.5, 0.8] \\ [0.4, 0.6] & - & [0.4, 0.5] & [0.5, 0.7] \\ [0.2, 0.5] & [0.5, 0.6] & - & [0.4, 0.5] \\ [0.2, 0.5] & [0.3, 0.5] & [0.5, 0.6] & - \end{pmatrix}$$

The new consensus levels are:

$$CL^1 = 0.911,\ CL^2 = 0.918,\ CL^3 = 0.906,\ CL^4 = 0.907.$$

Because all experts are over the minimum consensus threshold value $\gamma = 0.9$, the consensual collective IFRPR, from which the final solution of consensus will be selected, is computed.

5. Selection process of a GDM with IFRPRs

The selection process involves two different steps: (1) *aggregation* of individual preferences and (2) *exploitation* of the collective preference.

5.1. Aggregation of individual IFRPRs

In order to aggregate the individual preference relations, it is necessary to determine the weights associated to each expert. A general assumption in GDM problems is that the weights of experts is usually provided beforehand. However, this assumption may not be met in real situation, and therefore it is interesting to provide alternative ways to obtain that information. Trust is a key element of group negotiation, and as such we propose an SNA methodology for the representation and modelling of the trust relationships between experts in a group in GDM to allow to associate trust degree (TD) to experts (Section 3), which ultimately can be used as a reliable source of information to determine the experts' aggregation weights.

5.1.1. Aggregation based on consensus level and trust degree

In addition to the trust degree, the consistency degree or consensus level can also be regarded as reliable sources to derive experts' aggregation weights [7, 24, 25, 42, 43]. Since the sum of consensus levels are usually not equal to 1, the relative normalised consensus level are derived $RCL^h = \frac{CL^h}{\sum_{h=1}^{m} CL^h}$. The experts aggregation weights are obtained as a linear combination of their relative normalised consensus level (RCL) and trust degree (TD):

$$w^h = \beta \cdot RCL^h + (1 - \beta) \cdot TD^h \tag{30}$$

with $\beta \in [0,1]$ a parameter to control the degree of consensus and trust to implement in the aggregation of the group preferences.

Example 10. ***(Example 1 continuation).*** **Weighted Average Collective IFRPR.** Let $\beta = 0.5$, then experts' aggregation weights are:

$$w^1 = 0.29; \quad w^2 = 0.24; \quad w^3 = 0.25; \quad w^4 = 0.22.$$

The weighted average collective IFRPR is:

$$\widetilde{P}^c = \begin{pmatrix} - & [0.35, 0.55] & [0.40, 0.58] & [0.43, 0.64] \\ [0.45, 0.65] & - & [0.43, 0.61] & [0.50, 0.62] \\ [0.42, 0.60] & [0.39, 0.57] & - & [0.45, 0.58] \\ [0.36, 0.57] & [0.38, 0.50] & [0.42, 0.55] & - \end{pmatrix}$$

5.2. Exploitation of the collective IFRPR

Chiclana et al. [6] presented a quantifier guided non-dominance degree (QGNDD) method to derive a final ranking of the alternatives from a given FPR. This methodology is based on the use of the ordered weighted average (OWA) operators [52], which is guided by a linguistic quantifier [54] representing the concept of majority to implement in the decision making resolution [27]. Specifically, the linguistic quantifier is represented mathematically by a basic unit-monotonic (BUM) function $Q\colon [0,1] \to [0,1]$ such that $Q(0) = 0$, $Q(1) = 1$ and $Q(x) \geq Q(y)$ if $x \geq y$, which is used to compute the OWA operator weights as follows:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), i = 1, \ldots, n.$$

Yager [52] considered the parameterised family of regular increasing monotone (RIM) quantifiers $Q(r) = r^a$ $(a \geq 0)$ for such representation. This family of functions guarantees that: (i) all the experts contribute to the final aggregated value (strict monotonicity property), and (ii) associates, when $a \in [0,1]$, higher weight values to the aggregated values with associated higher importance values (concavity property) [24]. In particular, the value $a = 1/2$ is used to represent the fuzzy linguistic quantifier *'most of'*.

The use of the OWA operator involves the ordering of the values to aggregate from highest to lowest. Because interval numbers are not totally ordered, to extend the QGNDD to the case of the collective IFRPR, $\widetilde{P}^c = (\widetilde{p}^c_{ij})_{n\times n}$, we propose to apply the QGNDD to the following possibility degree matrix

$$PD = (pd_{ij})_{n\times n} = \Big(P(\widetilde{p}^c_{ij} \geq \widetilde{p}^c_{ji})\Big)_{n\times n}. \tag{31}$$

The quantifier guided non-dominance possibility degree associated to an alternative, $QGNDPD_i$, is defined as follows:

$$QGNDPD_i = \delta_Q(1 - \overline{p}^c_{ji}). \tag{32}$$

with $\overline{p}^c_{ji} = \max\{pd_{ji} - pd_{ij}, 0\}$, $pd_{ij} = P(\widetilde{p}^c_{ij} \geq \widetilde{p}^c_{ji})$ and δ_Q is an OWA operator guided by the linguistic quantifier represented by the BUM function Q.

Example 11. ***(Finishing Example 1).*** **Ranking of Alternatives.** The possibility degree (PD) matrix corresponding to the collective IFRPR is:

$$PD = \begin{pmatrix} - & 0.25 & 0.44 & 0.67 \\ 0.75 & - & 0.61 & 1.00 \\ 0.56 & 0.39 & - & 0.62 \\ 0.33 & 0.00 & 0.38 & - \end{pmatrix}.$$

The OWA operator weighting vector using the above linguistic quantifier *'most of'* is

$$W = (0.58, 0.24, 0.18)^T.$$

The quantifier guided non-dominance possibility degrees are:

$$QGNDPD = (0.88, 1.00, 0.13, 0.70).$$

The alternatives can be ranked from best to worst according to the degree up to which an alternative is not dominated by *'most of'* the rest of alternatives, resulting in x_2 being the solution of consensus of the GDM with IFRPRs:

$$x_2 \succ x_1 \succ x_4 \succ x_3.$$

6. Analysis of the Trust-Consensus Based Model

The trust-consensus model proposed in this paper presents the following main advantages and differences with respect to other consensus models proposed in the literature:

(1) It develops an interval-valued fuzzy SNA to represent the uncertainty or fuzziness of trust relationship between experts in a group, which allows:

   (a) The introduction of a formal definition of the concept of the trust degree (TD) of individual expert;

   (b) The TD is regarded as a reliable source of importance associated to experts in determining their aggregation weights.

   (c) The provision of an approach to resolve the unrealistic assumption of experts's weights to be known beforehand [26].

(2) It extends Tanino's multiplicative consistency property [38] from the case of FRPRs to IFRPRs, making possible:

   (a) To measure the consistency index (CI) for each one of the experts in the group;

   (b) To model the consensus level (CL) for IFRPRs taking into account both consistency (CI) and similarity (SI) criteria; and

   (c) To develop feedback mechanism to support those experts that are furthest from the group and therefore that are contributing less to consensus.

(3) It differs from the existing methods [5, 7, 42–45, 47, 49] in that two reliable resources, TD and CL, are used to derive the importance degree of experts in the proposed model. Indeed, as it was mentioned before, TD reflects the actual reputation of experts that derive from the trust relationship in the Social Network they are part of. TD is a priori knowledge to the decision making problem and it can be seen or classed as a subjective reliable source of information in deriving experts' weighting values. On the other hand, CL is a posteriori knowledge because it is directly computed once the experts provide their opinions on the decision making problem to solve. Thus, CL can be seen or classed as an objective reliable source of information in deriving experts' weighting values. This approach resembles the Bayesian approach in which the prior probability (knowledge) is used get the posterior probability (knowledge), and it aims to enhance the reliability of experts' weights in the group decision making process.

7. Conclusions

This paper presents a social network analysis (SNA) trust-consensus based model group decision making problems with interval-valued fuzzy reciprocal preference relation (IFRPR). The multiplicative consistency of FRPRs is also formally extended to the case of IFRPRs, and thus the consistency index (CI) of an IFRPR is introduced and analysed. By combining consistency (CI) and similarity (SI) criteria, a consensus level (CL) for IFRPRs is modelled. The experts' aggregation weights to compute the collective IFRPR are obtained by using experts' trust relationship, via its representation using an SNA methodology approach, which allows to formally define the concept of trust degree (TD) associated to each expert in the group, and their preferences relative normalised consensus levels. A feedback mechanism is also proposed to support those experts that are furthest from the group and therefore that are contributing less to consensus. It is important to remark that this model is one of the first efforts in combining TD and CL into the field of GDM problems. Finally, a quantifier guided non-dominance possibility degree (QGNDPD) based prioritisation method for IFRPRs is developed. It is worth remarking that in this paper, only direct trust relationship between group experts are used, and that other avenues to investigate in future include the use of indirect trust relationship by the trusted third parters (TTPs), which may be more complex and realistic than the direct one.

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